How the inclination angle affects the electrodynamics and statistics of radio pulsars

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• **Pulsar Magnetosphere**
  - Axisymmetric case
  - Orthogonal case
  - Inclined rotator

• **Statistics of Extinct Neutron Stars**
  - Evolution of the inclination angle
  - Extinct radio pulsars
  - Supersonic propeller
Pulsar Magnetosphere
1. Axisymmetric magnetosphere

• Explosion in the pulsar magnetosphere science (Contopoulos, Kazanas & Fendt, Gruzinov, Uzdensky, Mestel et al, Ogura & Kojima, Harding & Muslimov, Spitkovsky, Timokhin, Komissarov, McKinney, Arons et al, Istomin, Beskin & Nokhrina)

• In all the works an implicit assumption was done that the pair creation mechanism in polar region can support any current which is necessary to have MHD solution up to infinity.
1. Axisymmetric magnetosphere


- In all the works an implicit assumption was done that the pair creation mechanism in polar region can support any current which is necessary to have MHD solution up to infinity.
\[ W_{\text{tot}} \sim \frac{B_0^2 \Omega^4 R^6}{c^3} \]
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\[ j_{||} = \text{const} \]

\[ \Omega_F \neq \Omega \]
2. Orthogonal rotator

- Exact solution for zero longitudinal electric current
- Energy loss for local GJ current
- Position of the light surface
- PSR B1931+24
- Inclined split monopole
Orthogonal rotator, $I = 0$

$\chi = 90^\circ$

Beskin, Gurevich & Istomin (1983)
Orthogonal rotator, $I = 0$

Beskin, Gurevich & Istomin (1983)

Mestel, Panagi & Shibata (1999)
Orthogonal rotator, $I = 0$

- For $I = 0$ the toroidal magnetic field vanishes at the light cylinder.
- It means that for $I = 0$ there is NO ENERGY LOSS even for inclined rotator.
- All the energy loss is to be connected with the longitudinal current.
Orthogonal rotator - energy loss

Beskin, Gurevich & Istomin, JETP, 1983,
Beskin & Nokhrina, Astr. Lett, 2004

\[ \rho_{GJ} = -\frac{\Omega B}{2\pi c} \approx -\frac{\Omega B}{2\pi c} \left( \frac{\Omega R}{c} \right)^{1/2} \]

\[ W_{\text{tot}} = \Omega c^{-1} \oint [\mathbf{r} \times [\mathbf{J}_S \times \mathbf{B}]] \, dS \]
Orthogonal rotator - energy loss

Beskin, Gurevich & Istomin, JETP, 1983,

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\[ W_{\text{tot}} = \Omega c^{-1} \int [\mathbf{r} \times [\mathbf{J}_S \times \mathbf{B}]] \, dS \]

\[ W_{\text{tot}} = \frac{f^3}{64} \frac{B^2_0 \Omega^4 R^6}{c^3} \left( \frac{\Omega R}{c} \right) i_A \]

\[ i_A = \frac{j_\parallel}{\rho_{GJ} c} \]
Orthogonal rotator - energy loss


\[
W_{\text{tot}} = \frac{c}{4\pi} \int_{R_L} [E \times B] \, dS \approx \frac{c}{4\pi} E(R_L) B_{\phi}(R_L) R_L^2
\]

\[
B_{\phi}(R_L) \approx E(R_L) \frac{j_{||}}{\rho_{GJ} c}
\]

\[
W_{\text{tot}} \lesssim \frac{B_0^2 \Omega^4 R^6}{c^3} \left( \frac{\Omega R}{c} \right)^{1/2}
\]
Orthogonal rotator - energy loss


\[ W_{\text{tot}} \approx \frac{1}{4} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(1 + \sin^2 \theta \right) \]

It means that \( i_A \gg 1 \)
Orthogonal rotator - energy loss

- If $i_A \sim 1$, then $B_\phi << B_p$ on the light cylinder.
- It means that the light surface $|E| = |B|$ is to locate in the very vicinity of the light cylinder.
Inclined rotator – PSR B1931+24

- Switch-off – magnetodipole loss only
  \[ W_{\text{tot}} = \frac{1}{6} \frac{B_0^2 \Omega^4 R^6}{c^3} \sin^2 \chi \]

- Switch-on – current loss only \((f_* = 1.592 - 1.96)\)
  \[ W_{\text{tot}} = \frac{f_*^2}{8} \frac{B_0^2 \Omega^4 R^6}{c^3} i_0 \cos \chi \]

- The ratio
  \[ \frac{W_{\text{on}}}{W_{\text{off}}} = \frac{\dot{P}_{\text{on}}}{\dot{P}_{\text{off}}} = \frac{3 f_*^2}{4} \frac{i_0 \cos \chi}{\sin^2 \chi} \approx 1.5 \]
Inclined split monopole

For arbitrary inclination angle $\chi$

$$j_\parallel = \rho_{GJ}^{(0)} c$$

along the axis, so that

$$B_\varphi (R_L) \approx E (R_L)$$

Hence, incline split monopole cannot describe real dipole field of radio pulsars.
Statistics of Extinct Neutron Stars
Energy loss for $\chi < \pi/2$

**Magnetodipole loss**

$$W_{\text{tot}} = I\Omega \dot{\Omega} = \frac{1}{6} \frac{B_0^2 \Omega^4 R^6}{c^3}$$
Energy loss for $\chi < \pi/2$

**Magnetodipole loss**

$$W_{\text{tot}} = I\Omega \dot{\Omega} = \frac{1}{6} \frac{B_0^2 \Omega^4 R^6}{c^3} \sin^2 \chi$$

$$\Omega \cos \chi = \text{const}$$
Energy loss for $\chi < \pi/2$

**Magnetodipole loss**

$$W_{\text{tot}} = \frac{1}{6} \frac{B_0^2 \Omega^4 R^6}{c^3} \sin^2 \chi$$

$\Omega \cos \chi = \text{const}$

**Current loss**

$$W_{\text{tot}} = \frac{f^*}{8} \frac{B_0^2 \Omega^4 R^6}{c^3} i_0 \cos \chi$$

$\Omega \sin \chi = \text{const}$
• Time scale of the inclination angle evolution
  \[ \tau_\chi = \chi/\dot{\chi} \] is the same as \( \tau_\text{D} = P/\dot{P} \).

• Energy loss decreases mainly by the evolution of the inclination angle.

• Death line depends on the inclination angle.
Death line

Death line depends on the inclination angle
Death line

Gould

Rankin

New point

- Extinct radio pulsars – supersonic propeller transition line depends on the inclination angle as well.
- E.g., for magnetodipole energy loss

\[ P_{pr} = P_E \sin^{1/2} \chi \]

\[ P_E \approx 100 \frac{\mu_{30}^{1/2} c_{7}^{1/2} v_{7}^{1/2}}{(B_{ext})_{-6}^{1/2}} \text{ s} \]

(Lipunov et al, 1996)
Extinct radio pulsars (RS)

Beskin & Eliseeva, 2005

Radio pulsars
Extinct radio pulsars
Supersonic Propeller
Extinct radio pulsars (Arons)

Beskin & Eliseeva, 2005

Radio pulsars

Extinct radio pulsars

Supersonic Propeller
Supersonic propeller (RS)

\[ \cos \chi = \left( \frac{2\pi R}{cP} \right)^{1/2} \]
Supersonic propeller (Arons)

\[ N_3(P) = 1.2 \cdot 10^{-3} P E N^2 P^{-2} \int B_{12}^{2.74} \left( 1 + B_{12} \right)^{-3.7} \times \]

\[ \times \int_{0.185P^{0.8}_{E}}^{P} \frac{y}{P^{21/14}} \left( 1 + 0.1B_{12}^{1.7} \left( \frac{y}{P} \right)^{3.1} \right)^{0.9} dP \]
We predict for supersonic propellers

- $P \sim 6-12 \text{ s}$
- $P_{\text{dot}} \sim 10^{-13}$
- $N \sim 10^{-4} N_{\text{tot}}$
- Aligned or orthogonal

Radio transient sources?
Conclusion

Do not remember that radio pulsars are nonaxisymmetric objects.
For discussion

• Can the electric current be much larger/smaller that local GJ one?
• Whether the last observations of B1931+24 mean the hard evidence against the free particle escape from the NS surface.
• Is it possible to determine the evolution of the inclination angle?