Abstract

We study extremely nonlinear, coherent electromagnetic waves in the context of expanding plasma flows, where a confining external medium triggers the formation of a shock. Using a combination of analytical methods and Particle-In-Cell simulations, we look at mechanisms of wave generation and dissipation, as well as how the waves affect the particle distribution. For a large-amplitude waves of general polarization, any given set of wave parameters uniquely fixes the particle and energy flux associated with the flow. In cases where the wave properties can be constrained, this can be used to estimate the flow parameters. The prime application of our work is to pulsar winds and their termination shocks, where it provides a viable alternative to magnetohydrodynamic models.

The Wavelike Pulsar Wind

Pulsar winds are believed to carry most of their energy in the form of Poynting-flux close to the pulse, but to be particle-dominated at the termination shock (Kennel & Coroniti, 1984; Arons, 2002; Lyubarsky, 2003). The question of how the Poynting flux of the wind is dissipated has been subject to considerable debate in recent years. A central issue in the discussion is the fate of the highly nonlinear wave which is launched into the expanding wind by the spinning pulsar motion (i.e., in the general case where the neutron star spin and magnetic dipole axes are non-parallel). The behaviour of the wave properties can be constrained, this can be used to estimate the flow parameters. The prime application of our work is to pulsar winds and their termination shocks, where it provides a viable alternative to magnetohydrodynamic models.

Nonlinear TEM Waves

A nonlinear electromagnetic wave can propagate even if its frequency ω0 is less than the effective plasma frequency. For circularly polarized, purely transverse electromagnetic (TEM) waves in a cold plasma with a relativistic drift speed v∥ and a DC magnetic field B∥, both in the direction of wave propagation, the plane wave dispersion relation is

\[ \omega^2 = \frac{\omega_0^2}{1 - \frac{v^2}{c^2}} - \frac{k^2 c}{v^2} \]

where \( \omega = \omega_0 k \) is the index of refraction, \( \omega_0 = \sqrt{\varepsilon_0 \mu_0 c^2 n_e^2} \), \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability, respectively, n_e is the electron density, \( R = 1 - \frac{v^2}{c^2} \) is the Doppler factor, \( B = \varepsilon \mu_0 n_e m_e c \), and \( E \) is a dimensional amplitude defined by \( E = B_0 n_e m_e c \), where \( B_0 \) is the plasma electric field amplitude. In the unmagnetized limit, \( B_0 \rightarrow 0 \), the dispersion relation becomes (Akhiezer & Pokrovski, 1965)

\[ \omega = \frac{1}{\sqrt{\varepsilon_0 \mu_0 n_e^2}} \left( 1 + \frac{k^2}{\varepsilon_0 \mu_0 n_e^2} \right) \]

Application to the Crab

We find that (i) the coupling between the inner and outer parts of the Crab wind is nontrivial, and corresponds to a transition from subluminal to superluminal wave propagation; (ii) at the termination shock (Kennel & Coroniti, 1984; Arons, 2002; Lyubarsky, 1998). This modified scheme finds support in that the displacement current dominates the particle current by a large factor at large radii. We use analytical methods and Particle-In-Cell simulations to determine the state of the TEM wave as it propagates in the wind zone from \( r \approx r_{sh} \) to the termination shock and beyond.

Shock Jump Conditions

For the Crab pulsar wind termination shock, we analytically obtain \( \sigma \approx 9 \times 10^{13}, B^2 - 2.5, \) and \( \varepsilon = 9 \times 10^{-4} \), using canonical Crab parameters of \( \rho \) and \( n \) with units of mass density, \( \sigma \) the density, \( \varepsilon \) the particle current density, and the fact that the displacement current dominates the particle current by a large factor at large radii. The Shock jump parameters correspond to the rightmost state of each plot for best illustrating parameter dependencies, \( \omega/\omega_0 \) has been set artificially low here.

Decay of TEM Wave in Shock Interior

Fig 4. CIRCULARLY POLARIZED WAVE MODULATIONS OF SHOCK IN UNMAGNETIZED PLASMA (\( \sigma = 2.0 \) and \( B = 0 \)). Top row: Positron distribution in \( (x, u_x) \). Left: \( 10 < x < 20 \), right: \( 20 < x < 30 \). Bottom right: Local distribution of positrons in \( (x, u_x) \) in the precursor. Note the strong bunching of particles along the x-axis, as well as the oscillatory displacement from \( u_x = 0 \) induced by the wave. For the electrons, this displacement is opposite at \( x \).