Reconstruction of Interaction Sequences and Background Suppression in Advanced Compton Telescopes

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Compton imaging requires the measurement of:
- Gamma energy $E_\gamma$
- Energy transfer to electron $E_e$
- Direction and orientation of scattered $\gamma$–ray $\vec{n}_1$

**Compton kinematics:**

$$\cos \phi_E = 1 - \frac{m_e c^2}{E_\gamma - E_e} + \frac{m_e c^2}{E_\gamma}$$

$$\cos \phi_{geo} = \vec{n}_{src} \cdot \vec{n}_1$$

$\phi_{geo} = \phi_E$ defines an event circle about $\vec{n}_1$ for one photon.

**Imaging:** maximize likelihood

$$\prod_{i,j} p \left( \cos \phi_{geo, i,j}, \cos \phi_{E, i,tot, i} | E_\gamma, i, \vec{n}_{src, j} \right)$$

for multiple photons $i$ and sources $j$ (in principle). In practice ...

... that’s a different talk
Interaction sequence obtained by time–of–flight (TOF) measurement.

Advantage: clear separation of forward and backward events.
Disadvantage: low efficiency due to solid angle effect.
Compact Compton Telescopes

Approach: Increase efficiency by a factor up to 10 using a compact geometry:
- Single homogeneous detector: Xenon TPC, Ge or Si compact Compton telescopes
- Two–detector concept in compoact design (MEGA, Tiger)

Advantage: high detection efficiency
Disadvantage: time–of–flight unpractical (in most cases)
The Problem of the Time Sequence for Two-Site Events

Options:
- Time-of-flight
- Electron tracking
- Probabilities of sequences:
  - without assumption of source location: energy sharing.
  - with (some) assumption of source location, e.g., for a gamma-ray originating from the sky (above horizon):
    - locations of interactions
    - orientation
- Allow both sequences in imaging, assign corresponding probabilities in response function.
The Problem of the Time Sequence for Multiple-Site (>2) Events

Number of possible scatter directions and orientations grows with $n!$. Allowing any sequence is therefore *not* an option.

Options (other than TOF or tracking):

- Probabilities of sequences based on Compton kinematics, using redundant information on the interior angles.
- Probabilities of sequences based on additional criteria:
  - separations
  - energy deposit in "last" scatter
  - location of "first" scatter, ...
Test all $n!$ sequences for Compton kinematics.

$$\cos \phi_{E, 2} = 1 - \frac{m_e c^2}{E_3} + \frac{m_e c^2}{E_2 + E_3}$$

$$\cos \phi_{geo, 2} = \hat{n}_1 \cdot \hat{n}_2$$

Approaches:

Test statistics analogous to $\chi^2$:

$$T_c = \sum_i \frac{\left( \cos \phi_{E, i} - \cos \phi_{geo, i} \right)^2}{\sigma_{cE, i}^2 + \sigma_{cg, i}^2}$$

$$T_\phi = \sum_i \frac{\left( \phi_{E, i} - \phi_{geo, i} \right)^2}{\sigma_{E, i}^2 + \sigma_{g, i}^2}$$

applied in variations by several groups. (Aprile et al. ’94, Schmidt et al. ’99, Boggs & Jean ’00, Oberlack et al. ’00)

Works ok, but:

- Efficiency $\sim 50\%$ (detector–dep.)
- Little suppression of events that are uncontained or non–Compton
A Probabilistic Approach

How can one improve?

- The $\chi^2$-like test statistics is not optimal: it ignores the asymmetry of the underlying probability density function (pdf). Use the likelihood function and maximum likelihood ratio instead:

- If more than 1 sequence accepted apply additional criteria in a binary tree:
  - separations
  - energy deposit in "last" scatter
  - shortest leg is part of the sequence
Maximum Likelihood Ratio (MLR) Test with Probability Density Functions

\[ p_{geo} = p_E \left( \cos \phi_{geo} \mid \cos \phi_{geo, H}, r_{12}, \frac{r_{12}}{r_{23}} \right) \]

\[ p_E = p \left( \cos \phi_E \mid \cos \phi_{E, H}, E_2 + E_3 \right) \]

\[
\lambda = \frac{\max \left( (p_E * p_{geo}) \mid \cos \phi_{geo, H} = \cos \phi_{E, H} \right)}{\max \left( (p_E * p_{geo}) \mid \cos \phi_{geo, H}, \cos \phi_{E, H} \right)}
\]

- Compute \( \lambda \) for all sequences.
- \( \lambda \) is normalized: \( \max(\lambda) = 1 \)
- \( \lambda \) takes asymmetry of \( p_E \) into account.
Performance of MLR Test Statistic: Y–88 Simulation
Performance of MLR Test Statistic: Crab Nebula Simulation
• Significant reduction of false reconstructions with new algorithm
• Loss of good reconstructions because ambiguous sequences are rejected.
Tracing the Performance with Simulations

Incorr. Recons. Comp. Cont. Sec

Rejected Events Seq. 0, Compton Contained

Rejected NC Events – All Seqs. Rej: 0.58

Rejected Events Seq. 0
Tracing the Performance with Simulations

Without 'confusion':

- Incorr. Recons. Comp. Cont. Sec
  - Atten. length
  - Multiple seqs.
  - Min. Sep.
  - Shortest 3d sep.
  - Max E3
  - Min. Test. Stat
  - Total: 6.24

- Rejected Events Seq. 0, Compton Contained
  - Atten. length
  - Multiple seqs.
  - Min. Sep.
  - Shortest sep.
  - Max E3
  - Min. Test. Stat
  - Rejection Percentage: 0.70

- Rejected NC Events – All Seqs. Rej: 0.84
  - Atten. length
  - Multiple seqs.
  - Min. Sep.
  - Shortest sep.
  - Max E3
  - Min. Test. Stat
  - Rejection in %

- Rejected Events Seq. 0
  - Atten. length
  - Multiple seqs.
  - Min. Sep.
  - Shortest sep.
  - Max E3
  - Min. Test. Stat
  - Rejection Percentage: 0.74
Summary

- Compton Sequence Reconstruction (CSR) has to replace time-of-flight in various compact Compton telescopes.
- Current methods can be improved with statistical tools that take into account the asymmetric shape of underlying pdf’s.
- The new algorithm is still in its infancy, but shows promise:
  - Greatly improved discrimination against Non-Compton events (~80% rejection for LXeGRIT-type instrument)
  - Very small contamination with false reconstructions (~6%)
  - Clear discrimination between events with well-reconstructed sequences and events with ambiguous reconstructions (currently rejected).
- Reconstruction efficiency is still low (< 50%).
- Criteria/problems for detectors: spatial resolution, energy thresholds, energy resolution, number of scatters (Si only?), bremsstrahlung, pair production (high Z)
Outlook

- Implement "other" criteria as pdf's into MLR rather than using cuts (where appropriate and feasible).
- Study rejection power for multiple gamma-ray events (cascades etc.)
- Can we use polarisation in the CSR?
  - Even unpolarized incident gamma-rays are partially polarized after the first scatter! (Problem to be studied: how much? Bound electrons again...)
- Attempt to combine CSR and imaging?
  - CSR works great if source position known.
  - Polarisation criterion could be used to modulate ("reduce") event cone!
  - BUT: Will we be able to interpret the results and trace errors after combining two complicated problems?